

Spacecone quantization of AdS superparticle

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ABSTRACT

We first-quantize the superparticle describing free 10D IIB supergravity on $\text{AdS}_5 \times S^5$. We choose the worldline coordinate to be a combination of the bulk (spatial) coordinates of anti de Sitter space and the sphere. The Hamiltonian is *independent* of this “time” and the fermions. On the boundary, the representation of $\text{PSU}(2,2|4)$ becomes that of projective superspace. The prepotential propagator reproduces the known field-strength one.

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Introduction

The Green-Schwarz-style action for the superstring on $\text{AdS}_5 \times \text{S}^5$ [1] is complicated by its nonlinearity, particularly in fermions. In this paper we attack the simpler problem of the superparticle on the same space, which describes the superstring's ground state, 10D IIB supergravity. We choose a lightcone-type gauge that differs from the usual ones [2] in two ways:

- (1) The lightcone “time” is (complex) spatial [3], and comprised of “bulk” coordinates only, so as to manifestly preserve the $\text{SO}(3,1)$ Lorentz and $\text{SO}(4)$ internal symmetries of the boundary. It is also chosen to make the Hamiltonian time-independent.
- (2) The division of second-class fermionic constraints is such that the imposed constraints eliminate all fermions from the Hamiltonian. The surviving fermions thus appear trivially in the action, making the propagator (of the field/wave function) a δ -function in them.

The representation of the $\text{PSU}(2,2|4)$ symmetry generators, after solving the constraints and applying the corresponding gauge conditions, reduces to that of projective superspace (modified by trivial dependence on the ninth dimension [4]), plus time-dependent terms. The latter can also be found by applying time-dependence in the usual way through the Hamiltonian.

Lightcone

The general procedure for choosing lightcone gauges is to (1) determine everything in the action with a(n upper) minus (lightlike) index by the constraints, which are quadratic in covariant derivatives, and (2) gauge fix everything with a plus (canonically conjugate to the previous) using the gauge invariances generated by the same constraints, thus leaving only transverse degrees of freedom. In flat space, manifest Lorentz invariance is reduced from $\text{SO}(D-1,1)$ to $\text{SO}(D-2)$.

In the case of the superparticle, we can use the Casalbuoni-Freund-Brink-Schwarz action [5] without loss of generality, since all formulations lead to the same lightcone, where only physical degrees of freedom survive. The first-class constraints are

$$\begin{aligned} p^2 = 0 & \Rightarrow p^- = \dots, \quad x^+ = \tau \\ \gamma^+ \not{p} d = 0 & \Rightarrow \gamma^- d = \dots, \quad \gamma^+ \theta = 0 \end{aligned}$$

for which the quantity multiplying p^+ (which is assumed invertible) in the constraint is solved, and we have separated out the independent half of the κ constraints.

This still leaves the remaining second-class constraints

$$\gamma^+ d = 0, \quad \{\gamma^+ d, \gamma^+ d\} \sim \gamma^+ p^+$$

half of which can be separated according to some $U(1)$ charge (or discrete symmetry) as first-class. This half is then set to vanish, along with the conjugate half of $\gamma^- \theta$. In flat space, the $U(1)$ is chosen as one of the transverse (leaving p^+ invariant) Lorentz (rotation) $SO(2)$ generators, further reducing the manifest part of Lorentz invariance to $SO(D-4) \otimes SO(2)$.

Spacecone

For the AdS case it's useful to start with the parametrization of a group element of $PSU(2,2|4)$ as [6] (in the conventions of [4], which also contains a review)

$$g = \begin{pmatrix} I & w \\ 0 & I \end{pmatrix} \begin{pmatrix} u & 0 \\ 0 & \bar{u}^{-1} \end{pmatrix} \begin{pmatrix} I & 0 \\ -v & I \end{pmatrix} = \begin{pmatrix} u - w\bar{u}^{-1}v & w\bar{u}^{-1} \\ -\bar{u}^{-1}v & \bar{u}^{-1} \end{pmatrix}$$

where the symmetry group acts on the left and the gauge group on the right. The derivative form of the generators of these groups is then

$$G = g\partial_g = \begin{pmatrix} w\partial_w + u\partial_u & -w\partial_w w - u\partial_u w - w\partial_{\bar{u}}\bar{u} - u\partial_v\bar{u} \\ \partial_w & -\partial_w w - \partial_{\bar{u}}\bar{u} \end{pmatrix} \equiv \begin{pmatrix} G_u & -G_v \\ G_w & -G_{\bar{u}} \end{pmatrix}$$

$$D = \partial_g g = \begin{pmatrix} \partial_u u + \partial_v v & -\partial_v \\ \bar{u}\partial_w u + v\partial_u u + \bar{u}\partial_{\bar{u}}v + v\partial_v v & -\bar{u}\partial_{\bar{u}} - v\partial_v \end{pmatrix} \equiv \begin{pmatrix} D_u & -D_v \\ D_w & -D_{\bar{u}} \end{pmatrix}$$

(with “normal ordering” understood, so all derivatives act only on objects to the right of the generators).

We define p^+ by

$$D_u \approx D_{\bar{u}} \approx p^+ I$$

(for identity matrix “ I ”), which in turn defines how constraints are solved. (For example, pick only the half of the κ constraint that has p^+ in it. Note the sign in the definition of $D_{\bar{u}}$; the identity piece with opposite relative sign is the “S” part of PSU.) This effectively defines x^- as

$$sdet u = sdet \bar{u} = e^{x^-}$$

We then define x^+ so that the boundary limit $x^+ \rightarrow 0$ scales the two vertical halves of g (i.e., on the gauge group side) oppositely in the above decomposition. (Scaling them the same would be the “P” part of PSU.) This corresponds to

$$v \sim \sqrt{x^+}; \quad u, \bar{u} \sim (x^+)^{1/4}; \quad w \sim 1$$

(The boundary limit is the contraction of the gauge groups $\text{USp}(4) \rightarrow \text{I}[\text{USp}(2)^2]$, or $\text{SO}(5) \rightarrow \text{ISO}(4)$. In terms of the AdS/S radius, this is $x^+ \rightarrow R^2 x^+$, $R \rightarrow 0$. A perturbation expansion in R is also suggested by a random-lattice approach [7].)

Quantization

Besides the constraints/gauge invariances of flat space (Klein-Gordon equation and κ -symmetry), quadratic in covariant derivatives, we also have the linear ones $\text{USp}(2,2) \otimes \text{USp}(4)$ and “PS” (of PSU). We use all these, and a first-class half of the second-class constraints, to fix

<u>constraint</u>	<u>constrain</u>	<u>gauge away</u>
$USp's$	$p_v, \text{ some } p_u$	$x_v, \text{ some } x_u$
PS	$\text{some } p_u$	$\text{some } x_u$
p^2	p^-	x^+
$(pd)_u$	d_u	θ_u
d_v	d_v	θ_v

The quadratic constraints are parts of the matrix square D^2 [8]. For abbreviation in this table, we have used “ x ” for all bosons and “ θ ” for all fermions, and “ p ” and “ d ” for the corresponding covariant derivatives, with subscripts indicating whether they come from v or u (and \bar{u}). The p_u that are fixed (except p^- ; p^+ is not fixed) and all the d_v are fixed identically to 0; the other constrained covariant derivatives are determined in terms of the unconstrained ones, D_w and p^+ . Explicitly,

$$p_v = p_w \sim \sqrt{x^+} \partial_w, \quad p^- \sim \frac{p_w^2}{p^+} \sim x^+ \frac{\partial_x^2}{\partial_-}, \quad d_u \sim \frac{p_w d_w}{p^+} \sim x^+ \frac{\partial_x \partial_\theta}{\partial_-}$$

where we have indicated the x^+ (but not x^-) dependence.

All the coordinates that survive are w and x^- . (There is also x^+ , but it can be gauged to 0 in the Schrödinger picture, or fixed to τ in first-quantization.) These are exactly the coordinates of 4D N=4 projective superspace, plus the ninth dimension x^- , whose momentum p^+ counts the number of supergluons in the corresponding conformal field theory on the boundary [4], and thus should be invertible. These coordinates appear as (using Poincaré coordinates (x, x_0) and (y, y_0) for both AdS₅ and Wick-rotated S⁵)

$$w = \begin{pmatrix} y & \bar{\theta} \\ \theta & x \end{pmatrix}, \quad u = \bar{u} = \begin{pmatrix} \sqrt{y_0} I & 0 \\ 0 & \sqrt{x_0} I \end{pmatrix}$$

$$x^+ = x_0 y_0, \quad x^- = \ln \left(\frac{y_0}{x_0} \right) \quad \Rightarrow \quad p^+ \sim \partial_-, \quad p^- \sim x^+ \partial_+$$

(A similar choice of τ as a combination of an internal coordinate and a physical time-like coordinate has been applied to the pp-wave limit and its lightcone quantization [9].) Thus the Hamiltonian H appearing in the equation $i\partial_+ = H$ is independent of the “time” x^+ . In these coordinates the metric takes the form [4]

$$ds^2 = \frac{e^{-x^-} dy^2 - e^{x^-} dx^2 + dx^+ dx^-}{x^+}$$

Since the symmetry generators are x^+ -scale invariant (homogeneous of degree 0 in x^+) before imposing constraints, the only x^+ (or τ) dependence in them comes from solving the constraints, which mix the different sectors. The result is that these generators take the form of the projective superspace ones (including p^+ appearing as a charge), plus terms proportional to x^+ . (Projective superspace is defined by replacing all the constraints with simply $D_v = 0$, $D_u = D_{\bar{u}} = \partial_- I$.) Thus, in the Schrödinger picture the symmetry generators are identical to the projective ones, while in the Heisenberg picture they have linear time dependence, as can be generated by $e^{-ix^+ H}$.

Propagator

Since the Hamiltonian is independent of the fermions in the projective representation (i.e., the gauge $\theta_u = \theta_v = 0$), the propagator immediately follows from the bosonic result [10]:

$$\langle V(1)V(2) \rangle \sim \delta^8(\theta_1 - \theta_2)(s_x - s_y)^{-4}$$

$$s_x = \frac{(x_1 - x_2)^2 + (x_{10} - x_{20})^2}{2x_{10}x_{20}}, \quad s_y = \frac{(y_1 - y_2)^2 + (y_{10} - y_{20})^2}{2y_{10}y_{20}}$$

For comparison, we now rederive the propagator for the chiral field strength. The difference in division of second-class constraints (“chirality”), now distinguishing $SL(2, \mathbb{C})$ dotted (barred) and undotted spinors, is

$$d_v V = \bar{d}_v V = 0; \quad d_v \chi = \bar{d}_w \chi = 0, \quad \bar{d}_v \bar{\chi} = d_w \bar{\chi} = 0$$

where

$$\{d_v, d_w\} \sim \{\bar{d}_v, \bar{d}_w\} \sim p^+, \quad \{d_v, \bar{d}_w\} = \{\bar{d}_v, d_w\} = 0$$

$$\{d_v, d_v\} = \{d_v, \bar{d}_v\} = \{d_w, d_w\} = \{d_w, \bar{d}_w\} = 0$$

(10D “chirality” means depending only on the lightcone 8-spinor of positive $U(1)$ charge, namely $\theta_w \oplus \bar{\theta}_v$, in a lightcone representation, where the other half of the 16-spinor, $\theta_u \oplus \bar{\theta}_{\bar{u}}$, is again gauged away by κ symmetry.)

The solution to the reality condition on the chiral field strength is then given in terms of the prepotential by

$$d_w^4 \chi = \bar{d}_w^4 \bar{\chi} \quad \Rightarrow \quad \chi = \bar{d}_w^4 V, \quad \bar{\chi} = d_w^4 V$$

(There are also redundant reality conditions, $\bar{d}_v^4 \chi = d_v^4 \bar{\chi}$ and others from switching various numbers of d_w with \bar{d}_v .) This is essentially a Fourier transform in the fermions (up to powers of p^+), replacing $\bar{\theta}_w$'s in V with $\bar{\theta}_v$'s in χ . The field-strength propagator is then

$$\langle \chi(1) \bar{\chi}(2) \rangle = \bar{d}_{1w}^4 d_{2w}^4 \langle V(1) V(2) \rangle \sim (\hat{s}_x - \hat{s}_y)^{-4}$$

where \hat{s}_x and \hat{s}_y are the chiral-antichiral versions of s_x and s_y [11] (by adding fermion terms):

$$d_{1v} \hat{s}_{x,y} = \bar{d}_{1w} \hat{s}_{x,y} = \bar{d}_{2v} \hat{s}_{x,y} = d_{2w} \hat{s}_{x,y} = 0$$

(Chiral superspace corresponds to the “complex gauge” considered in [12]. It seems unlikely that a lightcone-type gauge based on chiral superfields could give a manifestly $SO(3,1)$ covariant formulation on the boundary, although it might relate to the 4D lightcone one.)

An analog of this construction is the corresponding propagator for 4D $N=1$ supersymmetry, where a chiral superfield can also be written in terms of a real prepotential [13] (although this is unnecessary for the construction). In an appropriate gauge,

$$\langle \phi(1) \bar{\phi}(2) \rangle = \bar{d}_1^2 d_2^2 \langle V(1) V(2) \rangle \sim \bar{d}_1^2 d_2^2 \delta^4(\theta_1 - \theta_2) x^{-2} = (x + i\theta_1 \bar{\theta}_2)^{-2}$$

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